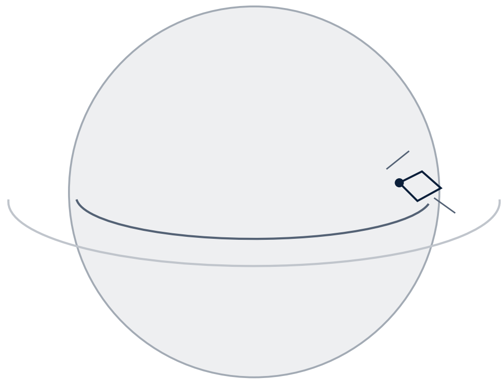

orbit as infrastructure

The First Harbor Above Earth

A Mathematical Feasibility Boundary for Cryogenic Refueling in Orbit

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Independent research draft, 2026

Draft v2

A civilization does not become spacefaring merely by touching space. It becomes spacefaring when space stops being a destination and becomes infrastructure. This paper studies one such piece of infrastructure: a cryogenic orbital propellant depot, or in ordinary language, a fuel harbor above Earth.

Version note

This v2 draft rewrites the earlier paper into a cleaner feasibility study, repairs spacing and readability, audits the numerical calculations, and separates the claim into explicit physical margins. It does not claim to be a flight design or a substitute for experimental validation. It claims something narrower: under stated assumptions, the main depot bottlenecks can be expressed as equations, and a plausible liquid oxygen baseline can satisfy them simultaneously.

Citation line

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Contents

1 Abstract

Reusable launch vehicles reduce the cost of reaching orbit, but they do not by themselves create a reusable space economy. A spacecraft that must carry every kilogram of propellant from Earth remains constrained by the same exponential burden: fuel is needed to move fuel. Orbital propellant depots change that architecture. They allow spacecraft to launch lighter, refill in orbit, and continue toward the Moon, Mars, or deep space.

This paper develops a mathematical feasibility boundary for an autonomous cryogenic orbital propellant depot. The objective is not to claim that every engineering problem is solved. The objective is to show how the main bottlenecks can be written as physical constraints and checked. The model combines orbital mechanics, heat transfer, cryogenic boil-off, pressure control, cryocooler power, solar-array sizing, transfer flow, chilldown, microgravity liquid acquisition, cavitation margin, slosh settling, and rocket-equation mission value.

A numerical liquid oxygen example is audited step by step. Under the stated assumptions, a 2 m-radius liquid oxygen tank has surface area $50.27\,\mathrm{m}^2$, volume $33.51\,\mathrm{m}^3$, an assumed heat leak of $10.05\,\mathrm{W}$ at $q'' = 0.2\,\mathrm{W/m}^2$, passive boil-off of $4.08\,\mathrm{kg/day}$, and zero-boil-off feasibility at approximately $235\,\mathrm{W}$ of idealized electrical cryocooler input, or $1.17\,\mathrm{kW}$ with a fivefold engineering margin. Including pump and control margins, the baseline requires approximately $10.46\,\mathrm{m}^2$ of solar array under the stated assumptions. The conclusion is that orbital depots are not forbidden by first-order physics. They are an engineering integration, validation, reliability, and scaling problem.

2 Why the problem matters

The first step into space was heroic. The next step must be logistical. NASA's LOXSAT mission description states that the Liquid Oxygen Flight Demonstration will test cryogenic fluid management technologies needed for in-space propellant depots, including boil-off reduction, transfer, tank-pressure maintenance, and propellant gauging [1]. NASA's Zero-Boil-Off work describes passive cryogenic storage as inadequate for long-duration missions and frames active cooling and mixing as a method for reducing or eliminating fuel loss while maintaining pressure control [2]. A 2024 review in *npj Microgravity* states that cryogenic on-orbit storage and refueling are necessary to extend, or even enable, deep-space exploration missions [3].

The real need is therefore not invented. What this paper tries to do is convert that need into a compact mathematical test. If a depot cannot close its heat, pressure, transfer, fluid acquisition, cavitation, and power margins, it fails. If it can close them together, the concept enters a feasible physical region.

3 Core claim

Let $u(t)$ be the depot control policy. In this simplified model, a cryogenic orbital propellant depot is feasible when there exists a policy $u(t)$ such that the following constraints remain true over the mission interval:

$$\boxed{\begin{aligned} \dot{Q}_{removed} &\geq \dot{Q}_{in}, \\ P(t) &< P_{max}, \\ \Delta P_{cap} &> \Delta P_{accel} + \Delta P_{flow}, \\ P_{line} - P_{sat}(T) &> \Delta P_{loss} + M, \\ P_{solar} &\geq P_{cryo} + P_{pump} + P_{control}. \end{aligned}} \tag{1}$$

Each line is a bottleneck:

- cooling must exceed heat leak,
- tank pressure must remain below its maximum safe limit,
- capillary liquid acquisition must beat microgravity acceleration and flow losses,
- transfer pressure must stay above saturation pressure with margin,
- generated and stored electrical power must cover cooling, pumping, and control.

The paper's contribution is not a finished vehicle. It is a feasibility boundary: a way to say exactly when the concept is inside or outside the region allowed by the simplified physics.

4 Notation

Symbol	Meaning	Unit
R	tank radius	m
A	tank surface area	m^2
V	tank volume	m^3
ϕ	fill fraction	dimensionless
q''	effective heat flux into tank	W/m^2
$\dot{Q}_{in}$	heat entering propellant system	W
$\dot{Q}_{removed}$	heat removed by active cooling	W
L_v	latent heat of vaporization	J/kg
P_{cryo}	electrical power required by cryocooler	W
COP	coefficient of performance	dimensionless
P_{max}	maximum allowed tank pressure	Pa
ΔP_{cap}	capillary pressure	Pa
ΔP_{loss}	flow pressure loss	Pa
M	required pressure margin against cavitation	Pa

5 Orbital mechanics: the depot's arena

For a circular orbit,

$$v_{orb} = \sqrt{\frac{\mu}{r}}, \qquad r = R_E + h, \tag{2}$$

where $\mu = GM_E$, R_E is Earth's radius, and h is depot altitude. The orbital period is

$$T_{orb} = 2\pi\sqrt{\frac{r^3}{\mu}}. \tag{3}$$

For $h = 400\,\text{km}$, $R_E = 6.371 \times 10^6\,\text{m}$, and $\mu = 3.986 \times 10^{14}\,\text{m}^3/\text{s}^2$,

$$r = 6.771 \times 10^6\,\text{m}, \tag{4}$$

$$v_{orb} = \sqrt{\frac{3.986 \times 10^{14}}{6.771 \times 10^6}} = 7.67 \times 10^3\,\text{m/s}, \tag{5}$$

$$T_{orb} = 5.54 \times 10^3\,\text{s} = 92.4\,\text{min}. \tag{6}$$

Thus the depot moves at nearly 7.7 km/s and experiences repeated sunlight/eclipses. A first-order average thermal load can be written

$$\langle \dot{Q}_{in} \rangle = f_{sun}\dot{Q}_{sun} + \dot{Q}_{Earth} + \dot{Q}_{internal}, \tag{7}$$

where f_{sun} is the sunlight fraction. A depot is not a static tank. It is a thermal machine moving through a cyclic environment.

6 Tank geometry and propellant mass

For a spherical tank,

$$A = 4\pi R^2, \qquad V = \frac{4}{3}\pi R^3. \tag{8}$$

Using a baseline radius $R = 2\,\text{m}$,

$$A = 4\pi(2)^2 = 50.27\,\text{m}^2, \tag{9}$$

$$V = \frac{4}{3}\pi(2)^3 = 33.51\,\text{m}^3. \tag{10}$$

With liquid oxygen density $\rho_{LOX} = 1141\,\text{kg/m}^3$ and fill fraction $\phi = 0.85$,

$$V_l = \phi V = 0.85(33.51) = 28.48\,\text{m}^3, \tag{11}$$

$$m_l = \rho_{LOX} V_l = 1141(28.48) = 3.25 \times 10^4\,\text{kg}. \tag{12}$$

This is not a small spacecraft tank. It is an illustrative depot-scale storage element. Tank wall mass, insulation mass, supports, ports, and safety systems are not included in this simple volume calculation.

7 Thermal bottleneck: boil-off

For a first-order feasibility model, heat leak is written

$$\dot{Q}_{in} = q'' A, \tag{13}$$

where q'' is the effective heat flux after insulation and shielding.

Assume

$$q'' = 0.2\,\text{W/m}^2, \qquad L_v = 213000\,\text{J/kg}. \tag{14}$$

Then

$$\dot{Q}_{in} = 0.2(50.27) = 10.05\,\text{W}. \tag{15}$$

Passive boil-off is

$$\dot{m}_{boil} = \frac{\dot{Q}_{in} - \dot{Q}_{removed}}{L_v}. \tag{16}$$

Without active cooling, $\dot{Q}_{removed} = 0$:

$$\dot{m}_{boil} = \frac{10.05}{213000} = 4.72 \times 10^{-5}\,\mathrm{kg/s}, \tag{17}$$

$$m_{boil,day} = (4.72 \times 10^{-5})(86400) = 4.08\,\mathrm{kg/day}. \tag{18}$$

Daily fractional loss is

$$\frac{4.08}{32499.99}(100) = 0.0125\%/\mathrm{day}. \tag{19}$$

The zero-boil-off condition is simply

$$\boxed{\dot{Q}_{removed} \geq \dot{Q}_{in}.} \tag{20}$$

This is the first bottleneck reduced to a gate. It does not say the insulation assumption is guaranteed. It says the design can be judged by a measurable margin.

Table 1: Sensitivity to effective heat flux for the same 2 m-radius LOX tank. Cryocooler power uses $COP = 0.0429$. Solar area uses fivefold thermal margin, $\eta_{solar} = 0.30$, $\eta_{sys} = 0.70$, and $f_{sun} = 0.62$.

q'' W/m^2	$\dot{Q}_{in}$ W	boil-off kg/day	P_{cryo} W	solar area, 5x m^2
0.2	10.05	4.08	235	6.62
1.0	50.27	20.39	1173	33.09
5.0	251.33	101.94	5860	165.47

The heat flux assumption dominates the thermal result. Therefore a serious version of this work must treat insulation performance as a sensitivity parameter, not as a hidden certainty.

8 Cryocooler power

Cryocooler electrical power is

$$P_{cryo} = \frac{\dot{Q}_{removed}}{COP}. \tag{21}$$

The Carnot upper limit is

$$COP_{Carnot} = \frac{T_c}{T_h - T_c}. \tag{22}$$

For liquid oxygen storage near $T_c = 90\,\mathrm{K}$ and a warm boundary near $T_h = 300\,\mathrm{K}$,

$$COP_{Carnot} = \frac{90}{300 - 90} = 0.429. \tag{23}$$

Let the real machine achieve 10% of Carnot:

$$COP = 0.10(0.429) = 0.0429. \tag{24}$$

For $\dot{Q}_{removed} = 10.05\,\mathrm{W}$,

$$P_{cryo} = \frac{10.05}{0.0429} = 235\,\mathrm{W}. \tag{25}$$

With a fivefold engineering margin,

$$P_{cryo,margin} = 5(235) = 1173\,\mathrm{W}. \tag{26}$$

The thermal problem therefore becomes an electrical-power and reliability problem. If heat leak rises fivefold, required cryocooler power rises fivefold. The relationship is linear and unforgiving.

9 Solar power and eclipse storage

Solar-array output averaged across sunlight and eclipse is approximated as

$$P_{solar} = \eta_{solar} S A_{solar} \eta_{sys} f_{sun}, \tag{27}$$

where $S = 1361\,\mathrm{W/m^2}$. Solving for solar area,

$$A_{solar} = \frac{P_{req}}{\eta_{solar} S \eta_{sys} f_{sun}}. \tag{28}$$

Let

$$\eta_{solar} = 0.30, \qquad \eta_{sys} = 0.70, \qquad f_{sun} = 0.62. \tag{29}$$

For the cryocooler margin alone,

$$A_{solar} = \frac{1173}{0.30(1361)(0.70)(0.62)} = 6.62\,\mathrm{m^2}. \tag{30}$$

Including pumping and control margin,

$$P_{req} = P_{cryo,margin} + P_{pump,margin} + P_{control}, \tag{31}$$

$$P_{req} = 1173 + 480 + 200 = 1853\,\mathrm{W}, \tag{32}$$

$$A_{solar} = \frac{1853}{0.30(1361)(0.70)(0.62)} = 10.46\,\mathrm{m^2}. \tag{33}$$

Eclipse battery energy is

$$E_{batt} = P_{req} t_{eclipse}. \tag{34}$$

For a 92.4 min orbit with $f_{sun} = 0.62$,

$$t_{eclipse} = (1 - 0.62)(92.4) = 35.1\,\mathrm{min} = 0.585\,\mathrm{h}, \tag{35}$$

$$E_{batt} = 1853(0.585) = 1084\,\mathrm{Wh}. \tag{36}$$

With usable battery specific energy $e_b = 180\,\mathrm{Wh/kg}$,

$$m_{batt} = \frac{1084}{180} = 6.0\,\mathrm{kg}. \tag{37}$$

With twofold margin,

$$m_{batt,margin} = 12\,\mathrm{kg}. \tag{38}$$

The simplified power system is therefore not massless, but it is not physically absurd. Real designs would add degradation, pointing, radiation, battery thermal management, power conversion losses, and redundancy.

10 Pressure bottleneck

Tank pressure is approximated by the ideal gas law:

$$PV = m_g R_s T. \tag{39}$$

Solving for maximum ullage gas mass,

$$m_{g,max} = \frac{P_{max} V_u}{R_s T}. \tag{40}$$

With ullage fraction $1 - \phi = 0.15$,

$$V_u = 0.15(33.51) = 5.03\,\mathrm{m}^3. \tag{41}$$

For oxygen gas, $R_s = 259.8\,\mathrm{J/(kg\,K)}$. Let

$$P_{max} = 300000\,\mathrm{Pa}, \qquad T = 90\,\mathrm{K}. \tag{42}$$

Then

$$m_{g,max} = \frac{300000(5.03)}{259.8(90)} = 64.5\,\mathrm{kg}. \tag{43}$$

If initial ullage gas mass is $m_{g,0} = 10\,\mathrm{kg}$, the allowable added gas mass is

$$\Delta m_g = 64.5 - 10 = 54.5\,\mathrm{kg}. \tag{44}$$

With passive boil-off of $4.08\,\mathrm{kg/day}$,

$$t_{vent} = \frac{54.5}{4.08} = 13.4\,\mathrm{days}. \tag{45}$$

With zero-boil-off control,

$$\dot{m}_{boil} = 0 \quad \Rightarrow \quad \frac{dP}{dt} \approx 0 \tag{46}$$

in the simplified model. The pressure bottleneck closes when

$$\boxed{P(t) < P_{max} \quad \forall t.} \tag{47}$$

This model ignores non-ideal gas behavior, stratification, sensor error, pressurant effects, vapor condensation dynamics, and tank structural details. It is a feasibility calculation, not a pressure-system design.

11 Transfer bottleneck

Suppose the depot transfers

$$m_{transfer} = 10000\,\mathrm{kg} \tag{48}$$

in

$$t_{transfer} = 2\,\mathrm{h} = 7200\,\mathrm{s}. \tag{49}$$

The required mass flow rate is

$$\dot{m} = \frac{10000}{7200} = 1.389\,\mathrm{kg/s}. \tag{50}$$

For a circular pipe of diameter $D = 0.05\,\mathrm{m}$,

$$A_p = \frac{\pi D^2}{4} = 0.001963\,\mathrm{m}^2. \tag{51}$$

Flow velocity is

$$v_f = \frac{\dot{m}}{\rho A_p} = \frac{1.389}{1141(0.001963)} = 0.620\,\mathrm{m/s}. \tag{52}$$

Pressure loss is estimated with Darcy-Weisbach:

$$\Delta P_{loss} = \left(f\frac{L}{D} + K \right) \frac{\rho v_f^2}{2}. \tag{53}$$

Let

$$f = 0.02, \qquad L = 10\,\mathrm{m}, \qquad D = 0.05\,\mathrm{m}, \qquad K = 5. \tag{54}$$

The dynamic pressure is

$$\frac{\rho v_f^2}{2} = \frac{1141(0.620)^2}{2} = 219\,\mathrm{Pa}. \tag{55}$$

The multiplier is

$$f\frac{L}{D} + K = 0.02\frac{10}{0.05} + 5 = 9. \tag{56}$$

Thus

$$\Delta P_{loss} = 9(219) = 1973\,\mathrm{Pa}. \tag{57}$$

Pump hydraulic power is

$$P_{pump} = \frac{\Delta P_{loss}\dot{V}}{\eta_p}, \qquad \dot{V} = \frac{\dot{m}}{\rho}. \tag{58}$$

Since

$$\dot{V} = \frac{1.389}{1141} = 0.001217\,\mathrm{m}^3/\mathrm{s}, \tag{59}$$

and $\eta_p = 0.5$,

$$P_{pump} = \frac{1973(0.001217)}{0.5} = 4.8\,\mathrm{W}. \tag{60}$$

This ideal hydraulic number is small. A real system must include valves, couplers, chilldown, transients, two-phase margin, sensors, controls, and redundancy. A one-hundredfold allowance gives

$$P_{pump,margin} = 480\,\mathrm{W}. \tag{61}$$

The flow bottleneck closes when

$$\boxed{\Delta P_{available} > \Delta P_{loss}.} \tag{62}$$

12 Chilldown bottleneck

Before cryogenic liquid enters a warm transfer line, the line must be cooled. Otherwise the propellant boils on contact. Chilldown energy is

$$Q_{chill} = m_{line} c_p (T_i - T_f). \tag{63}$$

Let

$$m_{line} = 10\,\mathrm{kg}, \quad c_p = 900\,\mathrm{J/(kg\,K)}, \quad T_i = 300\,\mathrm{K}, \quad T_f = 100\,\mathrm{K}. \tag{64}$$

Then

$$Q_{chill} = 10(900)(300 - 100) = 1.8 \times 10^6\,\text{J}. \tag{65}$$

The equivalent liquid oxygen boil-off is

$$m_{chill} = \frac{Q_{chill}}{L_v} = \frac{1.8 \times 10^6}{213000} = 8.45\,\text{kg}. \tag{66}$$

For a 10000 kg transfer,

$$\frac{m_{chill}}{m_{transfer}}(100) = \frac{8.45}{10000}(100) = 0.0845\%. \tag{67}$$

The chilldown bottleneck closes when

$$\boxed{\frac{m_{line} c_p (T_i - T_f)}{L_v m_{transfer}} \ll 1.} \tag{68}$$

This gives a design rule: make the line lightweight, pre-cool it if possible, and transfer enough propellant that chilldown loss is a small fraction of the operation.

13 Microgravity liquid acquisition

In gravity, liquid settles. In orbit, it does not. A depot must guide liquid to the outlet using surface tension, screens, vanes, or other propellant management devices.

Capillary pressure is

$$\Delta P_{cap} = \frac{2\sigma \cos\theta}{r_p}. \tag{69}$$

Acceleration pressure is

$$\Delta P_{accel} = \rho a L. \tag{70}$$

The liquid acquisition condition is

$$\boxed{\Delta P_{cap} > \Delta P_{accel} + \Delta P_{flow}.} \tag{71}$$

Using liquid oxygen surface tension $\sigma = 0.013\,\text{N/m}$, $\cos\theta = 1$, and pore radius $r_p = 5 \times 10^{-6}\,\text{m}$,

$$\Delta P_{cap} = \frac{2(0.013)}{5 \times 10^{-6}} = 5200\,\text{Pa}. \tag{72}$$

Let residual acceleration be

$$a = 10^{-4} g = 0.000981\,\text{m/s}^2, \qquad L = 2\,\text{m}. \tag{73}$$

Then

$$\Delta P_{accel} = 1141(0.000981)(2) = 2.24\,\text{Pa}. \tag{74}$$

Using $\Delta P_{flow} = 1973\,\text{Pa}$,

$$\Delta P_{accel} + \Delta P_{flow} = 1975\,\text{Pa}. \tag{75}$$

Therefore

$$5200 > 1975, \tag{76}$$

and the safety factor is

$$SF = \frac{5200}{1975} = 2.63. \tag{77}$$

For a smaller pore radius $r_p = 2 \times 10^{-6}$ m,

$$\Delta P_{cap} = 13000 \,\text{Pa}, \qquad SF = 6.58. \tag{78}$$

This does not design the screen. It shows that surface tension can exceed the modeled microgravity and flow losses by a useful margin.

14 Cavitation and two-phase margin

Liquid transfer fails if pressure drops below saturation pressure and vapor bubbles form. Define the subcooling pressure margin

$$M_{sub} = P_{line} - P_{sat}(T) - \Delta P_{loss}. \tag{79}$$

Safe transfer requires

$$M_{sub} > M, \tag{80}$$

where M is an engineering margin. Equivalently,

$$\boxed{P_{line} - P_{sat}(T) > \Delta P_{loss} + M.} \tag{81}$$

The controller must maintain

$$P_{line}(t) > P_{sat}(T(t)) + \Delta P_{loss}(t) + M \tag{82}$$

for the whole transfer interval. Cavitation is not defeated by optimism. It is defeated by pressure and temperature margin.

15 Slosh and settling

In microgravity, liquid motion can be slow. A simplified slosh frequency estimate is

$$\omega_s \approx \sqrt{\frac{g_{eff}}{L}}. \tag{83}$$

With $g_{eff} = 10^{-4} g = 0.000981 \,\text{m/s}^2$ and $L = 2$ m,

$$\omega_s = \sqrt{\frac{0.000981}{2}} = 0.0221 \,\text{rad/s}. \tag{84}$$

The slosh period is

$$T_s = \frac{2\pi}{\omega_s} = 284 \,\text{s} = 4.73 \,\text{min}. \tag{85}$$

A three-period settling wait is

$$t_{settle} = 3T_s = 14.2 \,\text{min}. \tag{86}$$

An autonomous controller can therefore include

$$\boxed{t_{wait} \geq N \frac{2\pi}{\omega_s}.} \tag{87}$$

This is a modest time cost compared with a multi-hour refueling operation, but real slosh is more complex than this simple estimate. Current experimental and numerical work shows that microgravity slosh remains a serious validation topic [3, 5].

16 Autonomous control law

The depot state can be written

$$x(t) = \begin{bmatrix} m_l(t) \\ m_g(t) \\ T_l(t) \\ T_g(t) \\ P(t) \\ \dot{Q}(t) \end{bmatrix}, \tag{88}$$

and the control input as

$$u(t) = \begin{bmatrix} P_{cryo}(t) \\ P_{pump}(t) \\ A_{valve}(t) \\ \dot{m}_{transfer}(t) \end{bmatrix}. \tag{89}$$

A model-predictive controller chooses

$$u^* = \arg\min_u J, \tag{90}$$

with cost

$$J = w_1 m_{lost} + w_2 \int_0^H (P - P_{target})^2 dt + w_3 t_{transfer} + w_4 \int_0^H P_{electric} dt + w_5 R_{cav}. \tag{91}$$

Lost propellant is

$$m_{lost} = \int_0^H \max\left(0, \frac{\dot{Q}_{in} - \dot{Q}_{removed}}{L_v}\right) dt. \tag{92}$$

Cavitation risk is penalized by

$$R_{cav} = \int_0^H \max\left(0, \Delta P_{loss} + M - [P_{line} - P_{sat}(T)]\right)^2 dt. \tag{93}$$

The controller is constrained by the five margins in Equation 1. Automation matters because the depot does not merely run pumps. It guards margins.

17 Mission value through the rocket equation

The depot's purpose is not elegance. It must buy mission capability. The rocket equation is

$$\Delta v = I_{sp} g_0 \ln\left(\frac{m_0}{m_f}\right). \tag{94}$$

Let

$$I_{sp} = 380\,\mathrm{s}, \qquad g_0 = 9.80665\,\mathrm{m/s^2}, \qquad m_f = 40000\,\mathrm{kg}, \qquad m_p = 100000\,\mathrm{kg}. \tag{95}$$

Then

$$m_0 = 140000\,\mathrm{kg}, \tag{96}$$

and

$$\Delta v = 380(9.80665)\ln\left(\frac{140000}{40000}\right) = 4668\,\mathrm{m/s}. \tag{97}$$

If 5000 kg of propellant is wasted,

$$m_0 = 135000\,\mathrm{kg}, \tag{98}$$

and

$$\Delta v_{loss} = 380(9.80665)\ln\left(\frac{135000}{40000}\right) = 4533\,\mathrm{m/s}. \tag{99}$$

The penalty is

$$\Delta v_{penalty} = 4668 - 4533 = 136\,\mathrm{m/s}. \tag{100}$$

In orbital mechanics, 136 m/s is not decorative. It may be the difference between healthy rendezvous margin and a compromised mission plan.

18 Feasibility score

Define five margins:

$$C_Q = \dot{Q}_{removed} - \dot{Q}_{in}, \tag{101}$$
$$C_P = P_{max} - P(t), \tag{102}$$
$$C_C = \Delta P_{cap} - \Delta P_{accel} - \Delta P_{flow}, \tag{103}$$
$$C_T = P_{line} - P_{sat}(T) - \Delta P_{loss} - M, \tag{104}$$
$$C_E = P_{solar} - P_{cryo} - P_{pump} - P_{control}. \tag{105}$$

A hard feasibility score is

$$F = \prod_{i\in\{Q,P,C,T,E\}} H(C_i), \tag{106}$$

where

$$H(C_i) = \begin{cases} 1, & C_i > 0, \\ 0, & C_i \le 0. \end{cases} \tag{107}$$

Thus $F = 1$ means all simplified bottlenecks are closed, and $F = 0$ means at least one fails.

For smoother optimization,

$$F_{smooth} = \prod_i \sigma(kC_i), \qquad \sigma(z) = \frac{1}{1+e^{-z}}. \tag{108}$$

This turns the depot from a vague idea into a map. Each design point either survives or dies by its margins.

19 Audited result table

20 Assumptions that matter most

A serious paper should not hide its weak points. The baseline is sensitive to:

- **Effective heat flux q''.** If heat leak rises from 0.2 to 5.0 W/m^2, required thermal power rises by 25x.
- **Cryocooler performance.** The assumed 10% of Carnot is a simplified value, not a selected flight machine.
- **Tank geometry.** A sphere minimizes surface area for volume, but real spacecraft packaging may force other shapes.

Table 2: Simplified bottleneck closure for the baseline LOX example. Values are feasibility calculations, not flight specifications.

Bottleneck	Closure condition	Baseline result
Orbit	$v = \sqrt{\mu/r}$	$v = 7.67\,\mathrm{km/s}$, $T = 92.4\,\mathrm{min}$
Tank geometry	$A = 4\pi R^2$, $V = 4\pi R^3/3$	$A = 50.27\,\mathrm{m}^2$, $V = 33.51\,\mathrm{m}^3$
Boil-off	$\dot{Q}_{removed} \geq \dot{Q}_{in}$	$\dot{Q}_{in} = 10.05\,\mathrm{W}$, passive boil-off 4.08 kg/day
Cryocooler power	$P_{cryo} = \dot{Q}/COP$	235 W, or 1173 W with 5x margin
Solar power	$P_{solar} \geq P_{req}$	$10.46\,\mathrm{m}^2$ with eclipse factor and margins
Pressure rise	$P(t) < P_{max}$	passive vent time 13.4 days; ZBO gives bounded pressure in model
Transfer flow	$\Delta P_{available} > \Delta P_{loss}$	$\Delta P_{loss} = 1973\,\mathrm{Pa}$
Chilldown	$m_{chill}/m_{transfer} \ll 1$	0.0845% for stated line and transfer mass
Microgravity acquisition	$\Delta P_{cap} > \Delta P_{accel} + \Delta P_{flow}$	safety factor 2.63 at $r_p = 5\,\mu\mathrm{m}$
Cavitation	$P_{line} - P_{sat} > \Delta P_{loss} + M$	must be maintained by pressure and temperature control
Slosh	$t_{wait} \geq N2\pi/\omega_s$	14.2 min for three periods
Mission value	rocket equation	5000 kg loss costs about 136 m/s

- **Microgravity liquid acquisition.** The capillary calculation proves a pressure margin can exist, not that the selected screen/PMD is buildable.
- **Transfer losses.** The pipe model is idealized and does not include every coupler, valve, transient, thermal, and two-phase effect.
- **Long-duration reliability.** Years of operation require radiation tolerance, fault handling, materials validation, docking reliability, and sensor stability.

These are not fatal to the paper. They are the exact places a v3 model or simulator should explore.

21 What this paper does not prove

A mathematical feasibility boundary is not a flight qualification. This paper does not solve seal wear, docking risk, contamination, valve lifetime, long-term sensor drift, radiation hardening, multi-vehicle operations, cryogenic material fatigue, thermal-structural coupling, regulation, or operations. It also does not replace microgravity testing.

What it does show is narrower and useful: the central physical bottlenecks can be reduced to equations, and a plausible baseline case can close them under explicit assumptions. The remaining uncertainty is not whether first-order physics permits a depot. The uncertainty is whether engineering can hold those margins for years, repeatedly, with real vehicles attached.

22 Next work

The next version should add:

1. a small simulator that sweeps tank radius, fuel type, heat flux, solar area, and transfer mass;
2. plots of boil-off, pressure margin, solar area, and feasibility score;
3. methane and hydrogen cases alongside LOX;

4. higher-fidelity thermal radiation and eclipse modeling;
5. a more serious PMD/screen model for microgravity acquisition;
6. comparison to public CFM and ZBO test data where available.

The PDF is the argument. The simulator would be the proof engine.

23 Conclusion

There are moments in civilization when the next frontier is not reached by one heroic machine, but by a quiet piece of infrastructure. Ports mattered more than ships. Rail mattered more than locomotives. Power grids mattered more than individual generators. If humanity is to become a true spacefaring civilization, orbit must become infrastructure.

This paper modeled one candidate infrastructure: an autonomous cryogenic orbital propellant depot. The core feasibility result is

$$\boxed{\begin{aligned} \dot{Q}_{removed} &\geq \dot{Q}_{in}, \\ P(t) &< P_{max}, \\ \Delta P_{cap} &> \Delta P_{accel} + \Delta P_{flow}, \\ P_{line} - P_{sat}(T) &> \Delta P_{loss} + M, \\ P_{solar} &\geq P_{cryo} + P_{pump} + P_{control}. \end{aligned}} \tag{109}$$

When these inequalities are true, the depot lies inside a feasible physical region. In the audited LOX example, the thermal, pressure, transfer, capillary, power, and mission-value calculations all close under the stated assumptions.

The strongest conclusion is not that orbital fuel depots are simple. They are not. The conclusion is that the path is visible. The bottlenecks are mathematical. The margins can be measured. The harbor above Earth is not a fantasy. It is an engineering frontier waiting to be made reliable.

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